Overview

⇒ Representation and Reasoning System
• Syntax of Datalog
• Semantics of Datalog
• Adding Variables to Semantics
• Models
• Logical Consequence
• Two Views of Semantics
• Introduced a task domains: robot delivery and wiring

• Introduced the symbolic approach
  - Symbols have meaning to the knowledge engineer
  - Symbols used to build a knowledge base that computer is told about
    + Facts about the world
    + Rules about the world
  - Computer reasons with the facts and rules to make new conclusions
A Representation and Reasoning System (RRS) is made up of:

- Formal language (syntax):
  + Specifies the legal sentences (the range of things that can be said)

- Semantics:
  + Specifies the meaning of the symbols (for your domain)
  + Specifies what is a correct conclusion

- Reasoning theory or proof procedure:
  + Specification of how an answer can be produced
  + Can be nondeterministic

Implementation of an RRS:

- Reasoning procedure
  + Resolves nondeterminism of reasoning theory
Different RRS’s

• Different RRS’s
  - With different syntaxes
    + Actually different connectors: ways to build complex expressions
  - Or with different semantics for connectives

• Different RRS’s good for different domains

• The richer the syntax, the more difficult the reasoning procedure

⇒ Choose the simplest RRS possible for your application
Simplifying Assumptions of Initial RRS

• An agent’s knowledge can be usefully described in terms of individuals and relations among individuals
• An agent’s knowledge base consists of definite and positive statements
• The environment is static
• Only a finite number of individuals of interest in the domain
• Each individual can be given a unique name
⇒ Datalog
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Syntax of Datalog

• Variable
  - starts with upper-case letter

• Constant
  - starts with lower-case letter or is a sequence of digits (numeral)

• Predicate symbol
  - starts with lower-case letter

• Term
  - either a variable or a constant

• Atomic symbol (atom)
  - of the form \( p \) or \( p(t_1, \ldots t_n) \) where \( p \) is a predicate symbol and \( t_i \) are terms
More Syntax of Datalog

• Definite Clause
  - either an atomic symbol (a fact) or of the form
    \[ a \leftarrow b_1 \land \ldots \land b_m \]

• Query
  - of the form \(?b_1 \land \ldots \land b_m\)

• Knowledge Base
  - set of definite clauses

⇒ Syntax allows us to write sentences about the world
  - Whether sentences are true or not depends on what the symbols mean,
    which will be specified by the semantics
Example

• Knowledge base
  \(\text{male(\text{william})}\)
  \(\text{male(\text{george})}\)
  \(\text{female(\text{sally})}\)
  \(\text{father(\text{william, george})}\)
  \(\text{father(\text{george, sally})}\)
  \(\text{person}(X) \leftarrow \text{female}(X)\)
  \(\text{person}(X) \leftarrow \text{male}(X)\)
  \(\text{parent}(X, Y) \leftarrow \text{father}(X, Y)\)
  \(\text{grandfather}(Z, X) \leftarrow \text{father}(Z, Y) \land \text{parent}(Y, X)\)

• What are the constants?
  - What are the predicate symbols?
  - What are the variables?
  - Whether knowledge base is correct depends on semantics
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Semantics

Semantics concerns two things

• Set of individuals in the domain, and relations between them
  - What individuals and relations you choose depends on what you want to reason about
  - Individuals could even be abstract things like colors, if that is what you want to reason about

• How constants and predicate symbols in the syntax correspond to the individuals and relations in the domain

We call this an interpretation:

• A domain, and a mapping from the syntax to the domain
An interpretation is a triple $I = (D, \phi, \pi)$ where

- $D$ the domain, is a nonempty set. Elements of $D$ are individuals
- $\phi$ maps each constant to an element of $D$
  - Constant $c$ denotes individual $\phi(c)$.
- $\pi$ maps each n-ary predicate symbol to subset of $D^n$
  - Alternatively, can think of $\pi$ as mapping each tuple $D^n$ to true or false
  - **NOTE:** it does not map it to a subset of constants$^n$
    - Common mistake, don’t make it on your homework
Example Interpretation

• D is the set of people
  William, George, Sally
  - It is the actual people, not the names

• \( \phi \) maps constants of syntax
to objects in the domain
  \( \phi(william) = \text{William} \)
  ...

• Knowledge Engineer decides D
  and mapping of all constants to D
Example Continued

• William and George are male, Sally is female

• Lets have $\pi$ map

  $\text{male}$ to $\{<\text{William}>, <\text{George}>\}$

  $\text{female}$ to $\{<\text{Sally}>\}$

• Knowledge Engineer decides on mapping of predicates
  - Must decide on the mapping for all predicates
  - Hence, must do mapping for $\text{male}$, even if no facts in $KB$ about $\text{male}$

• This is an example of an intended interpretation:
  - The interpretation that the knowledge engineer has in mind when coming up with language and knowledge base
• Example: (focus on all interpretations, not just intended one)
  - Language with constants $a$ and $b$ and 1-ary predicate $female(\_)$
  - Domain with $D = \{x, y, z\}$
  - How many different $\phi$’s?
Second Example

• Example: (focus on all interpretations, not just intended one)
  - Language with constants $a$ and $b$ and 1-ary predicate $female(\_)$
  - Domain with $D = \{x, y, z\}$
  - How many different $\phi$’s?

<table>
<thead>
<tr>
<th></th>
<th>$\phi_i(a)$</th>
<th>$\phi_i(b)$</th>
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</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
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<td>$\phi_2$</td>
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<td>$\phi_7$</td>
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<td>$\phi_8$</td>
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<tr>
<td>$\phi_9$</td>
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</tr>
</tbody>
</table>

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Example Continued

• How many π’s?
Example Continued

• How many $\pi$’s?

<table>
<thead>
<tr>
<th>$x \in \pi_i(female)$</th>
<th>$y \in \pi_i(female)$</th>
<th>$z \in \pi_i(female)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_1$</td>
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<tr>
<td>$\pi_2$</td>
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<td>$\pi_7$</td>
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<tr>
<td>$\pi_8$</td>
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</tbody>
</table>

• How many different interpretations are there altogether (different combinations of $\phi$ and $\pi$)?
Determining Truth of Ground Atoms in I

- Ground atom has no variables
- \( p(t_1, ..., t_n) \) maps to true if \( (\phi(t_1), ... \phi(t_n)) \in \pi(p) \) otherwise to false
- What does \( \text{male}(\text{george}) \) map to?
  - \( \phi(\text{george}) = \text{George} \)
  - \( \pi(\text{male}) = \{<\text{William}>, <\text{George}>\} \)
  - \( <\text{George}> \in \{<\text{William}>, <\text{George}>\} \)
  - So it maps to true

- For predicates without arguments
  \( \pi(p) \) is either the set with the empty tuple \( \{<>\} \) or it is empty \( {} \)

\( \Rightarrow \) Semantics of Ground Atoms comes from interpretation
Semantics of Connectives

• Still need to specify what ‘∧’ and ‘←’ mean

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ∧ q</th>
<th>p ← q</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
<td>true</td>
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<tr>
<td>true</td>
<td>false</td>
<td>false</td>
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<tr>
<td>false</td>
<td>false</td>
<td>false</td>
<td>true</td>
</tr>
</tbody>
</table>

- Nota bene!
  + p←q is true when both p and q are false
  + p∧q doesn’t always correspond to ‘english’ meaning

• Thus h ← b₁ ∧ ... ∧ bₘ is false in interpretation I
  if h is false in I and each bᵢ is true in I

⇒ Semantics of ‘∧’ and ‘←’ part of Datalog
Example

• Is \( \text{male}(\text{george}) \land \text{female}(\text{sally}) \) true in \( I \)?

• Is \( \text{male}(\text{george}) \leftarrow \text{female}(\text{sally}) \) true in \( I \)?

• Is \( \text{male}(\text{george}) \leftarrow \text{female}(\text{william}) \) true in \( I \)?

• Is \( \text{female}(\text{george}) \leftarrow \text{male}(\text{william}) \) true in \( I \)?
Limitations of Datalog

male(george).
female(sally).
...

- Even if every object is male or female, both predicates needed
  - Datalog does not include an operator that means negation
- Cannot write a rule that ensures just one of male and female is true for any person
  - Up to knowledge engineer to ensure each person is just one of them
  - More expressive formalisms can handle this (negative knowledge)
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How do we interpret clauses such as 
\[ \text{person}(X) \leftarrow \text{female}(X) \]

Clause is true if it is true for all values of \( X \)
- \( \text{person}(X) \) must be true whenever \( \text{female}(X) \) is true
- Remember, knowledge engineer had to specify mapping for all predicates, even room
  - \( \pi(\text{female}) \subseteq \pi(\text{person}) \)

It really has a universal quantifier
- For all \( X \) \( \text{female}(X) \leftarrow \text{person}(X) \)

So, variables have an implicit universal quantifier over the clause
Variable Assignment: Formal Definition

- Define a variable assignment $\rho$
  - Maps each variable to some object in the domain
- Together $\rho$ and $\varphi$ assign each term to some object in the domain
- Together $\rho$ and interpretation $I$ map every clause to true or false
  + Even ungrounded ones
- Now we can say:
  - A clause is true in an interpretation if it is true for all variable assignments
Example

• Interpretation I
  - $\pi(male) = \{<William>, <George>\}$
  - $\pi(female) = \{<Sally>\}$
  - $\pi(person) = \{<William>, <George>, <Sally>\}$

• Are the following true?
  \[
person(X) \leftarrow \text{male}(X)
  \]
  \[
person(X) \leftarrow \text{female}(X)
  \]
  \[
\text{male}(X) \land \text{female}(X)
  \]
  \[
\text{male}(X) \lor \text{female}(X)
  \]
  \[
person(X) \leftarrow \text{female}(X) \land \text{male}(\text{william})
  \]
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Sets of Clauses

• A set of clauses is true in an interpretation if each clause is true in the interpretation
  - Note that we universally quantify for the variables over each clause
  - In other words, if two clauses use the same variables, it is the same as if they used different variables

\[
\begin{align*}
\text{person}(X) & \leftarrow \text{male}(X) \\
\text{parent}(X,Y) & \leftarrow \text{father}(X,Y) \\
\text{grandfather}(Z,X) & \leftarrow \text{father}(Z,Y) \land \text{parent}(Y,X)
\end{align*}
\]
Models

• A model of a set of clauses is an interpretation in which all the clauses are true
  - Start with KB and look at what interpretations can be true

• Example $KB$:
  
  $p \leftarrow q$.
  
  $q$.

\[
\begin{array}{|c|c|c|c|}
\hline
I_1 & \pi(p) & \pi(q) & \pi(p \leftarrow q) & \text{Model of KB?} \\
\hline
\text{TRUE} & \text{TRUE} & \text{FALSE} & \text{TRUE} & \text{FALSE} \\
\text{FALSE} & \text{FALSE} & \text{TRUE} & \text{FALSE} & \text{FALSE} \\
\hline
\end{array}
\]
Example with constants

- Example: (focus on all interpretations, not just intended one)
  + Language with constants $a$ and $b$ and 1-ary predicate $girl(_)$
  + Domain with $D = \{x, y, z\}$
  + 9 $\phi$’s and 8 $\pi$’s, so 72 interpretations

- How many models of $KB = \{girl(a), girl(b)\}$?
  (Checking each would take too long, so let’s break down into subcases)
  - Case 1: $\phi_i(a) = \phi_i(b)$
    + How many of the 9 $\phi_i$’s have $\phi_i(a) = \phi_i(b)$
    + When $\phi_i(a) = \phi_i(b) = x$, which $\pi_i$’s make $KB$ true?
    + So how many models with $\phi_i(a) = \phi_i(b)$
  - Case 2: $\phi_i(a) \neq \phi_i(b)$
    + How many of the 9 $\phi$?
    + When $\phi_i(a) = x$ and $\phi_i(b) = y$, which $\pi$’s make the $KB$ true?
    + So how many models with $\phi_i(a) \neq \phi_i(b)$?
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⇒ Logical Consequence

- Two Views of Semantics
Logical Consequence

- If $KB$ is a set of clauses and $g$ is a conjunction of atoms, $g$ is a logical consequence of $KB$, written $KB \models g$, if $g$ is true in every model of $KB$.
  - This tells us that our $KB$, by its definition, always forces $g$ to be true
  - Other terms that mean same thing:
    - $g$ logically follows from $KB$
    - $KB$ entails $g$

- That is, $KB \models g$ if there is no interpretation in which $KB$ is true and $g$ is false.

- $KB \not\models g$ if $g$ is not a logical consequence of $KB$
Example Revisited

- \( KB: \)
  - \( p \leftarrow q. \)
  - \( q. \)

<table>
<thead>
<tr>
<th></th>
<th>( \pi(p) )</th>
<th>( \pi(q) )</th>
<th>( \pi(p \leftarrow q) )</th>
<th>model of KB?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 )</td>
<td>TRUE</td>
<td>TRUE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_2 )</td>
<td>TRUE</td>
<td>FALSE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_3 )</td>
<td>FALSE</td>
<td>TRUE</td>
<td></td>
<td></td>
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<tr>
<td>( I_4 )</td>
<td>FALSE</td>
<td>FALSE</td>
<td></td>
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</tr>
</tbody>
</table>

- Does \( KB \models p? \)
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⇒ Two Views of Semantics
User’s View of Semantics

• Choose a task domain: intended interpretation
• Associate constants with individuals you want to name
• For each relation you want to represent, associate a predicate symbol in the language
• Tell the system clauses that are true in the intended interpretation: *axiomatizing the domain*
  - hopefully you tell it enough knowledge about the domain so that it can conclude everything you want it to
• Ask questions about your domain
Computer’s view of semantics

• Computer given the knowledge base
  - Computer doesn’t have access to the intended interpretation

• User asks it a question $g$
  - Computer should answer true if $KB \models g$
    - $g$ is true in all models, so is true in user’s intended interpretation
  - Otherwise, computer should answer “I don’t know”
    - There is at least one model in which $g$ is false

+ Note $g$ might have been true in user’s intended interpretation. In this case, user
didn’t have enough clauses in the KB to sufficiently narrow down the models

• Aside: computer could answer the question by enumerating over all of the possible interpretations (model checking)
  - But number of interpretations grows quickly!!
Summary of Semantics

• User has intended interpretation
  But just tells the computer a small set of facts that hopefully adequately captures the user’s intended interpretation

• Computer answers true if all interpretations that make KB true (models) make the question true
  - Now we have specs for the computer’s reasoning algorithm
  - It should answer yes if $KB \models q$, other answer don’t know