Overview

⇒ Lowest-Cost-First

• Best-First Search
• A* Search
• Iterative Deepening

Lowest-cost-first Search

• Sometimes there are costs associated with arcs. The cost of a path \( g \) is the sum of the costs of its arcs
• Lowest-cost-first search finds the shortest path to a goal node
• Frontier is implemented as a priority queue ordered by \( g \)
  - At each stage, it selects the shortest path on the frontier
• When arc costs are equal ⇒ breadth-first search
Heuristic Search

• Previous methods do not take into account goal until at goal node
• Often there is extra knowledge that can be used to guide the search: heuristics
• Use \( h(n) \) as estimate of distance from node \( n \) to a goal node
• \( h(n) \) is underestimate if it is less than or equal to the actual cost of the shortest path from node \( n \) to a goal
• \( h(n) \) uses only readily obtainable information about a node
Best-first Search

- Idea: always select node on the frontier with smallest $h$-value
- Treat the frontier as a priority queue ordered by $h$
- Uses space exponential in path length

Applying Best-First Search to Top-Down Proofs

- How could we use this in searching through resolutions?

```
?a ∧ d
h ← m. p. 
g ← f.  k ← m.
f ← p.  g ← m. 
d ← p.  f ← m. 
c ← f.  d ← m. 
n ← m.  m ← d. 
m ← c.  j ← c. ∧ a. ∧ d.
a ← g.  b ← j. 
b ← k.  d ← m. 
a ← b ∧ c.  a ← g. 
```
Seems Like A Good Idea But ...

- Not guaranteed to find a solution, even if one exists

- It doesn’t always find the shortest path

Example with Top-Down Theorem Proving

- Not guaranteed to find a solution, even if one exists

```g ← a
  g ← d ∧ e
  a ← b
  b ← a
  d
e
  ? g```
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A* Search

- A* search takes path to a node and heuristic value into account
  - $g(n)$ be the cost of the path found to node $n$
    + From lowest-cost first search
  - $h(n)$ be the estimate of the cost from $n$ to goal
    + From best-first search
  - Let $f(n) = g(n) + h(n)$.
    + $f(n)$ is estimate of path from start to goal via $n$
    \[
    \begin{array}{c}
    \text{start} \quad \xrightarrow{\text{actual}} \quad n \quad \xrightarrow{\text{estimate}} \quad \text{goal} \\
    \hline
    g(n) \quad \hline
    h(n) \quad \hline
    f(n)
    \end{array}
    \]
- A* orders the frontier by $f(n)$
  + Stops when min node in frontier is goal node
A* Finds Optimal Solution

- If there is a solution, A* always finds an optimal solution
  - the first path to goal that it finds is optimal
- If...
  - the branching factor is finite (not necessarily a finite number of nodes)
  - arc costs are bounded above zero
    (there is some $\epsilon > 0$ such that all of the arc costs are greater than $\epsilon$)
  - $h(n)$ is an underestimate of the cost of the best path from $n$ to a goal node and $\geq 0$

Proof that if it finds a path, the path is optimal

- Let an optimal path have weight $f^*$
- Cells in the frontier are ordered by $g(n) + h(n)$
  - Where $g(n)$ is strictly increasing as you go down the path
  - And $h(n)$ is a lower-estimate $\geq 0$ of the remaining distance
- Assume $A^*$ stops at a goal node with non-optimal path $p$
  - So, $p$ was on top of the frontier
  - Since $p$ is not optimal, $g(p) > f^*$
  - Since $p$ ends at the goal $g(p) = f(p)$, and so $f(p) > f^*$
  - But, part of the optimal path will be in the frontier,
    and it will have an $f$-value $\leq f^*$ (since $f$-values never over estimate)
  - Hence, it would have been higher in the frontier than $p$,
    and so $p$ would not have been chosen
Summary of Search Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Frontier Selection</th>
<th>Halts?</th>
<th>Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth-first</td>
<td>Last node added</td>
<td>No</td>
<td>Linear</td>
</tr>
<tr>
<td>Breadth-first</td>
<td>First node added</td>
<td>Yes</td>
<td>Exp</td>
</tr>
<tr>
<td>Best-first</td>
<td>Global min $h(n)$</td>
<td>No</td>
<td>Exp</td>
</tr>
<tr>
<td>Lowest-cost-first</td>
<td>Global min $g(n)$</td>
<td>Yes</td>
<td>Exp</td>
</tr>
<tr>
<td>$A^*$</td>
<td>Global min $f(n)$</td>
<td>Yes</td>
<td>Exp</td>
</tr>
</tbody>
</table>

Proof that it will find a path

- Let an optimal path have weight $f_1$
- Only a finite number of subpaths $m$ have $g$-score $\leq f_1$
  - Because each arc has weight at least $\epsilon$ and finite branching
  - Note: subpath might not end at a goal node and $g$-score measures the full cost of the subpath
- So, finite number of subpaths $n \leq m$ have $f$-score at most $f_1$
  - Because $f$-score of subpath is greater than its $g$-score
- A subpath of the optimal path is always in frontier and its $f$-score always at most $f_1$
- After at most $n$ steps, optimal path must be on top of frontier (if we haven’t stopped earlier)
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Iterative Deepening

- So far all search strategies that are guaranteed to halt use exponential space
- Idea: let’s recompute elements of the frontier rather than saving them
- Look for proofs of depth 0, then 1, then 2, then 3, etc
- You need a depth-bounded depth-first searcher
- If proof cannot be found at depth B, look for proof at depth $B + 1$
Depth-bounded depth-first search

- $\text{dbsearch}(N,D,P)$ is true if $P$ is path of length $D$ from $N$ to goal

```
dbsearch(Node,0,[Node]) :-
    is_goal(Node).

dbsearch(Node,D,NewP) :-
    D > 0,
    neighbors(Node,Neighbors),
    member(NewNode,Neighbors),  \(\Leftarrow\) non deterministic
    D1 is D - 1,
    dbsearch(NewNode,D1,P),
    NewP = [Node|P].
```

? dbsearch(start,5,Path).

- A bit different from previous versions
  - Gathers up all neighbors, and then non-deterministically chooses one
  - Builds the path on the way out