Question 1  (1 marks)
Definition of a DFA: A DFA is a 5 tuple \( \{Q, \Sigma, \delta, q_0, F\} \)
where \( Q \) is a set of states,
\( q_0 \) is the start state and \( q_0 \in Q \)
\( F \) is the set of accept states and \( F \subseteq Q \)
and \( \delta : Q \times \Sigma \rightarrow Q \)

Give the definition of an NFA. You only have to state what is different. State your answer using mathematical notation (or in words).

Question 2  (1 marks)
Here is the definition of computation for a DFA.
Let \( M = \{Q, \Sigma, \delta, q_0, F\} \) be a DFA and let \( w \) be a string where \( w = w_1...w_n \) and \( w_i \in \Sigma \).
\( M \) accepts \( w \) if
(1) there is a sequence of states \( r_0...r_n \)
(2) \( r_0 = q_0 \)
(3) \( r_n \in F \)
(4) \( r_i = \delta(r_{i-1}, w_i) \)

Give the definition for computation for a NFA. You only have to state what is different. State your answer using mathematical notation (or in words).
Question 3  (1 marks)
Give the definition of a regular language. Start with “A language $L$ is regular if ...”

Question 4  (1 marks)
Draw a DFA for $\{w\mid$ the second last character of $w$ is a 1$\}$. Explain what each state means.

Question 5  (1 marks)
Draw a NFA for $\{w\mid$ the third last character of $w$ is a 1$\}$. Make sure your NFA is using non-determinism effectively. Explain how the non-determinism is being used.
**Question 6** (1 marks)

Give a regular expression for the language \( \{ w \mid \text{the third last character of } w \text{ is a } 1 \} \). (All strings in this language will be at least 3 characters long.)

**Question 7** (2 marks)

Let \( L \) be a regular language, and \( N \) an NFA that recognizes it. Construct an \( M \) NFA that recognizes \( L^* \). Explain the construction in both words and mathematical notation.

What does your construction show about regular languages?
Question 8  (3 marks)
Convert the following from a NFA to a regular expression using the method given in class. Show all steps.

\[
\begin{array}{c}
\text{start} \\
\rightarrow \\
a \\
\rightarrow 0 \\
\rightarrow 0 \\
\rightarrow b \\
\end{array}
\]
Context Free Languages

Question 9  (3 marks)

Let $L = \{w#y \mid \text{the } i\text{th character in } w \text{ is not the same as the } i\text{th character in } y \text{ for some } i \text{ and } |w| \geq i \text{ and } |y| \geq i\}$. $\Sigma = \{0, 1\}$. Give a PDA for this language. You can describe the PDA in words.
Question 10  (2 marks)

$L = \{ w#y \text{ such that } |w| \neq |y| \}$. $\Sigma = \{0,1\}$. Give a grammar that recognizes this language.

Question 11  (2 marks)

Let $L = \{ w \mid vw \in A \}$ where $A$ is context free and $\Sigma = \{0,1\}$. ($L$ is the suffix of $A$). Prove that $L$ is also context free.
Question 12 (2 marks)
Let $L = \{ w | w \in \{a, b, c\}^* \text{ and number of a’s, b’s and c’s is the same} \}$. Prove that $L$ is not context free.

Turing Machines

Question 13 (1 marks)
Define what configuration $uaq,bv$ means, in terms of what state the TM is in, what is on the tape, and where the tape head is.
Question 14  (1 marks)
If a TM is in configuration 0011q2001, what will be the configuration if it makes the transition $\delta(q_2, 0) \rightarrow (q_3, x, L)$.

Question 15  (1 marks)
Define what it means for a TM machine $T$ to accept a string $w$, in terms of configurations. Your definition should apply to both deterministic and non-deterministic machines.

Question 16  (1 marks)
What is the difference between a Turing recognizable language and a Turing decidable language?