CS533 Final Exam

Name: _______________________

Question 1 (2 marks)
Sketch the argument that for a non-deterministic TM, one can create a deterministic TM that recognizes the same language.

Answer:

Question 2 (1 marks)
What does it mean for a non-deterministic TM that is a decider to accept a string, versus rejecting a string. Make use of the term ‘computation path’ in your explanation. Does it make sense to ask whether a non-deterministic TM that is a recognizer rejects a string?

Answer: For a NTM: if any computation path accepts, the NTM accepts.

For a NTM that is a decider, it rejects if no all computation paths reject, or in other words, no computation path accepts.

For a NTM that is a recognizer, there might be no path that accepts, but there might be some paths that end in a reject, and some paths that loop forever. In this case, we cannot say that it rejects, just that it does not accept.

Question 3 (1 marks)
What is the definition of a mapping reduction?

Answer:

Question 4 (1 marks)
What is the definition of a polynomial time mapping reduction?

Answer:

Question 5 (1 marks)
Define $A_{CFG}$ using only mathematical notation, no English words.

Answer: $A_{CFG} = \{ (G, w) | w \in L(T) \}$

Question 6 (2 marks)
Show that $A_{CFG}$ is decidable. Give a TM that can decide this.

Answer:

Question 7 (3 marks)
A linear bounded automata is a type of Turing machine which can only use the tape that the input was written on. It’s tape alphabet is allowed to include extra characters not used in the input.

Show that $E_{LBA}$ is not decidable. You will need to do a reduction from $A_{TM}$ via accepting configuration histories of problems in $A_{TM}$.
Answer:

$A_{TM}$ can be viewed as accepting configuration history. For $\langle T, w \rangle$, it is in $A_{TM}$ if there is an accepting configuration of $T$ on $w$. We can build a LBA that only accepts accepting configurations for $\langle T, w \rangle$. So, we can convert a problem from $A_{TM}$ into a problem of $E_{LBA}$. Note that we are actually mapping from $A_{TM}$ to $E_{LBA}$. Since we are just interested in undecidability, it does not matter. But we would not be able to set up a mapping reduction.

Question 8  (1 marks)
Here is a proof that uses a reduction to show that $E_{TM}$ is not decidable.

| Assume that $E_{TM}$ is decidable. Let $R$ be a TM that decides it.
| Construct TM $S$ to decide $A_{TM}$ as follows:
| $S$ = “On input $\langle M, w \rangle$ where $M$ is a TM, and $w$ is a string
| 1. Construct a description of a TM $M_1$
| $M_1$ = “On input $x$:
| (a) If $x \neq w$, reject
| (b) If $x = w$, run $M$ on input $w$ and accept if $M$ does
| 2. Run $R$ on input $\langle M_1 \rangle$
| 3. If $R$ accepts, reject; if $R$ rejects, accept

Explain why this cannot be turned into a mapping reduction.

**Answer:** Cannot flip accepts and rejects.

Question 9  (1 marks)
In reference to the above question, what related problem can be turned into a mapping reduction?

**Answer:** $\overline{E_{TM}}$

Question 10  (2 marks)

Prove that if $A$ is undecidable, so is $\overline{A}$. Use a proof by contraction and a construction. Clearly show what your assumption is, and clearly state why there is a contraction, and what that implies. Also, prove that your construction is correct. This is an straightforward proof, but make sure you are precise and rigorous in your proof.

**Answer:** Assume that $\overline{A_{TM}}$ is a decidable.

Let $M$ be a decider for that language.

Build $N$ that will decide $A$ as follows.

$N$: on input $w$
run $M$ on $w$
if $M$ rejects, accept, else reject

Proof that $N$ decides $A$:

$N$ is a decider since it just runs $M$ which is also a decider

$N$ accepts $w$ exactly when $M$ rejects $w$

So $N$ decides $\overline{L(M)} = \overline{A} = A$ So $N$ decides $A$. Contraction since $A$ is undecidable.

So our assumption must be false.

Question 11  (2 marks)

Here is another reduction from class
Prove $\text{REGULAR}_{TM}$ is undecidable.

Assume $\text{REGULAR}_{TM}$ is decidable. Let $R$ decide it. Construct TM $S$ to decide $A_{TM}$ as follows:

$S = \text{"On input } \langle M, w \rangle \text{ where } M \text{ is a TM, and } w \text{ is a string}

1. Construct a description of a TM $M_2$
   $M_2 = \text{"On input } x:\n   (a) \text{ If } x \text{ is of the form } 0^n1^n, \text{ accept}
   (b) \text{ Otherwise, run } M \text{ on input } w \text{ and accept if } M \text{ does}
2. Run $R$ on input $\langle M_1 \rangle$
3. If $R$ accepts, accept; if $R$ rejects, reject

Reframe the answer to use a mapping reduction. Clearly state what the mapping reduction is mapping from and to.

**Answer:** let $f$ map from $A_{TM}$ to $\text{REGULAR}_{TM}$. $f: \langle T, w \rangle$

Create the TM $M_2$  Output $M_2$

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**Question 12** (2 marks)

Give an algorithm to sort a sequence of strings (you can assume that $>$) will give you the comparison for 2 strings. Analyze its running time with respect to the number of strings.

**Answer:**

**Question 13** (3 marks)

Prove that a language can be decided by a non-deterministic TM in polynomial time iff there is a polynomial time verifier for it. Note: this is an ‘if and only if’, meaning that there are two parts to it.

**Answer:**

**Question 14** (1 marks)

What is the definition of NP-complete.

**Answer:**