Chapter 3

Question 1: Exercise 3.7

There are a few issues with this proof. Explain what each issue is, and whether it can be fixed.

Question 2: Problem 3.9

Part a
Give a language that can be recognized by a 2-PDA that cannot be recognized by a 1-PDA. Give a brief explanation of how the 2-PDA will recognize the language.

Part b.0
Before proving part b, first prove that a 2-PDA can simulate a Turing machine by doing the following:

- Explain how the two stacks can used to represent a configuration $uqv$ of a TM.
- Explain how to represent the initial configuration of the TM (where its inpt is on the tape, and the tape head is on the first square of the tape).
- Explain how the 2-PDA will simulate the TM when it transitions to the left $\delta(q_1, a) = (q_2, b, L)$. You do not need to account for how the TM acts at the beginning of the tape.
- Explain how the 2-PDA will simulate the TM when it transitions to the right $\delta(q_1, a) = (q_2, b, R)$. You must account for how the TM acts at the end of the tape.

Part b
Now that you have proved that a 2-PDA can simulate a TM, give a short explanation, in terms of the Church-Turing hypothesis, of why a 3-PDA cannot be more powerful than a 2-PDA.

Question 3: Problem 3.15 Part b, c, d

Use the same style of answer as the textbook gives for part a. Specify whether you are using a deterministic or non-deterministic TM.

Question 4: Problem 3.18

Modify the proof of theorem 3.21. You just need to give the constructions. You do not have to prove that the constructions are correct.

For construction of the enumerator $E$, you can assume that $E$ can come up with the strings of $\Sigma^*$ in lexicographical order: $s_1, s_2, s_3, \ldots$. Also keep the construction of $E$ as simple as possible: do you really need the double loop that was needed in theorem 3.21.

You can assume that the language is infinite. But explain why this assumption is needed.
Question 5: Problem 3.12

For this problem, just show that you can simulate a TM using a TM using a left-reset machine. In other words, you just need to do one part of the ‘iff’.

Describe how the machine works at the tape-head level. In particular, describe what the reset machine will need to do for any transition of the TM that moves the tape head left. Do not worry about the case where the TM moves left at the beginning of the tape.

Chapter 4

Before starting this homework, look at the problems that have answers (marked with an ‘A’). Spend some time on each one to see if you can figure out how to do it before looking at the answer. This should help prepare you to do the homework questions.

Where you need to prove that something is decidable, you need to construct a TM. You can use the TMs that were constructed in the proofs in Chapter 4 as a substep of the TM that you create.

Question 6: Exercise 4.2

In addition to giving the construction, also prove that your construction is correct: that the TM is a decider and that it decides the desired language. You just need to give the proof of the construction for this question, and any questions where it is specifically asked.

Question 7: Exercise 4.3

Question 8: Exercise 4.4

This is the language of grammars, in which each grammar accepts the empty string, as well as other possible strings.