Chapter 1: Pumping Lemma

Question 1: Problem 1.39

First, think of a language \( L \) that requires \( k \) states in a DFA, and where there is a string \( w \in L \) where \( |w| \geq k \).

Formally give the language, and informally describe the DFA that will recognize it.

Prove that it cannot be done in \( k - 1 \) states. Your proof will be similar to the proof of the pumping lemma.

Question 2: Problem 1.54

Chapter 1: Closure

Question 3: Problem 1.43

Question 4

Pick one of the following two questions.

Option 1:

This is a variation on question 1.45.

Define \( A/B = \{ w | wx \in A \text{ for some } x \in B \} \).

The textbook asks you to prove that if \( A \) is regular, and \( B \) is any language, then \( A/B \) is regular.

I am going to have you do a variation of this. Show that if \( A \) and \( B \) are regular, so is \( A/B \).

Do this as a proof by construction. You do not have to give a formal definition for the delta transition, but you do need to explain the operation of your constructed FA in enough detail that someone could write a formal definition. Make sure you explain what your states are, start state and end state.

Hint: your construction will use a DFA for \( A \) and \( B \).

Option 2:

Question 1.57

This question also involves a construction.

Chapter 2: CFG

Question 5: Question 2.2

Part 0

Prove that CFLs are closed under union.
Part 1
Give an example of two CFL’s whose intersection is a CFL.

Let $A = \emptyset$. Let $B = \Sigma^*$. 
A and $B$ are both regular, and therefore CFL.
Since they are regular, the intersection is also regular, and so a CFL.

Part a
There is a quick solution to this. As the textbook says, you can use the fact that $a^n b^n c^n$ is not a CFL, without proving it. But you need to prove that $A$ and $B$ are CFL. Make sure you fully write out the proof.

Also explain why this is not a contraction with part 1.

Part b
From part a, use the actual languages $A$ and $B$, together with Theorem 0.20. You might find it easy to use proof by contradiction.

To format complementation in latex, use $\overline{A}$

Question 6: Question 2.4 Part b, c, e, f

Question 7: Question 2.6 Part b, d

For part b, think of several cases. Here are the cases that I can think of. If you can think of others, please write them down explicitly.
Case 1: $a^i b^j$ where $i > j$
Case 2: $a^i b^j$ where $i < j$
Case 3: any string that starts with $b$.
Case 4: any string of the form $a^i b^j a \{a, b\}^*$

For part d, I am changing the language to the following:
$\{x_1 \# x_2 \# \ldots \# x_k | k \geq 2, \text{each } x_i \in \{a, b\}^*, \text{and for some } i \text{ and } j, \text{ where } i \neq j, x_i = x_j^R\}$
This way, you do not have to worry about the case where $i = j$, thus making $x_i$ a palindrome.

I just used 3 variables for my answer.

Question 8: Question 2.14