Exact Matching, Part 1

photophosphorescent

Steven Bedrick
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Plan for today:

String-matching problems

Exact matching: naïve algorithm

Exact matching: linear-time algorithm

The exact string-matching problem:

Given a short pattern P...

... and a longer text T...

... identify the locations in T where instances of P occur.

Often, we have a large number of patterns...

... and a large amount of text...

... and we want to find instances efficiently.

Worst-case: $O(nm)$

Ideally: sub-linear

Approximate matching:

Like exact matching, but allowing “small differences”:

Functionally similar amino acid substitutions/indels?

Vowel shifts, systematic consonant dropping, morphological variants, etc.?

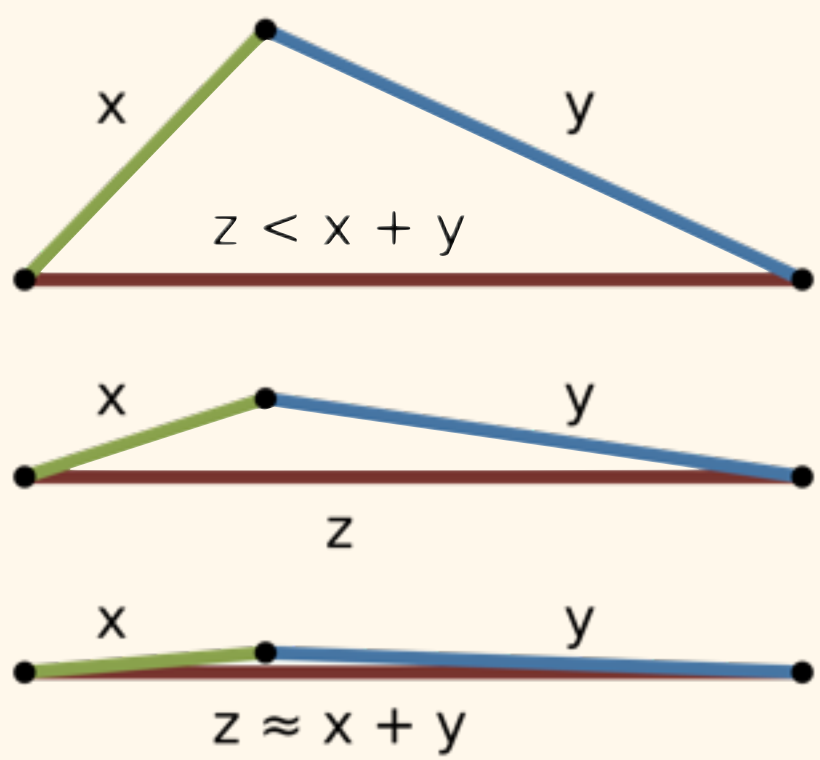
Review: distance metrics:

1. Non-negative property:

   $d(\mathbf{a}, \mathbf{b}) \geq 0$ for all **a** and **b**

2. Zero property:

   $d(\mathbf{a}, \mathbf{b}) = 0$ iff **a** = **b**



3. Symmetry:

   $d(\mathbf{a}, \mathbf{b}) = d(\mathbf{b}, \mathbf{a})$ for all **a** and **b**

4. Triangle inequality:

   $d(\mathbf{a}, \mathbf{b}) + d(\mathbf{b}, \mathbf{c}) \geq d(\mathbf{a}, \mathbf{c})$ for all **a**, **b** and **c**

http://en.wikipedia.org/wiki/Triangle_inequality

Approximate matching:

Precise formulation of “distance” will is heavily application-dependent…

… but the algorithms don’t care!

Today, though, we're talking about exact.

P = "xtpxtd"

T = "xluxtpxtdqwtdxtpxtsyxtpxtdy"

Occurrences of P in T:

xlu<u>xtpxtd</u>qwtdxtpxtsy<u>xtpxtd</u>y

Demo

Performance:

If P has length m, and T length n, worst-case is $O(mn)$.

- suppose P is *aaaaab*,
  and T is *aaaaaaaaaaaaaaaa*

Don't use the naïve algorithm!

Performance:

How might we improve?

P = "xtpxtd"

T = "xluxtpxtdqwtdxtpxtsyxtpxtdy"

Does our pattern have any internal structure?

Demo

Improvements to exact match almost always involve pre-processing either P or T.

The goal: save time (comparisons) by identifying repetitive elements.

The choice of which to focus on is application dependent:

Mapping new DNA sample? Probably transform P, since you'll be working with sequence-tagged sites.

Of course, there are probably errors in your STS library, so...

The choice of which to focus on is application dependent:

Searching for arbitrary words in a large text? Focus on transforming the text.

Today's algorithm will focus on patterns.

Definitions: for a string S:

$S[i,j]$ = contiguous substring starting at i and ending at j. $S(i) = S[i,i]$

S = aardvark

S[2,4] = ard   S(4) = d

For $i > 1$, $Z_i(S)$ is the length of the longest prefix of $S[i,|S|]$ that is also a prefix of S.

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S = xtpxtd

$S[4,|S|]$ = xtd

$Z_4(S) = 2$

For $i > 1$, $Z_i(S)$ is the length of the longest prefix of $S[i,|S|]$ that is also a prefix of S.

$S = \underline{xt}pxtd$

$S[4,|S|] = \underline{xt}d$

$Z_4(S) = 2$

In this string, for all other positions k, $Z_k(P) = 0$.

P = aardvark

$$Z_2(P) = 1$$

P = aardvark

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P = aardvark

$Z_2(P) = 1$

$Z_6(P) = 1$

P = alfalfa

$Z_4(P) = 4$

P = aardvark

$Z_2(P) = 1$

$Z_6(P) = 1$

P = alfalfa

$Z_4(P) = 4$

P = photophosphorescent

$Z_6(P) = Z_{10}(P) = 3$

P = aardvark

$Z_2(P) = 1$

$Z_6(P) = 1$

P = alfalfa

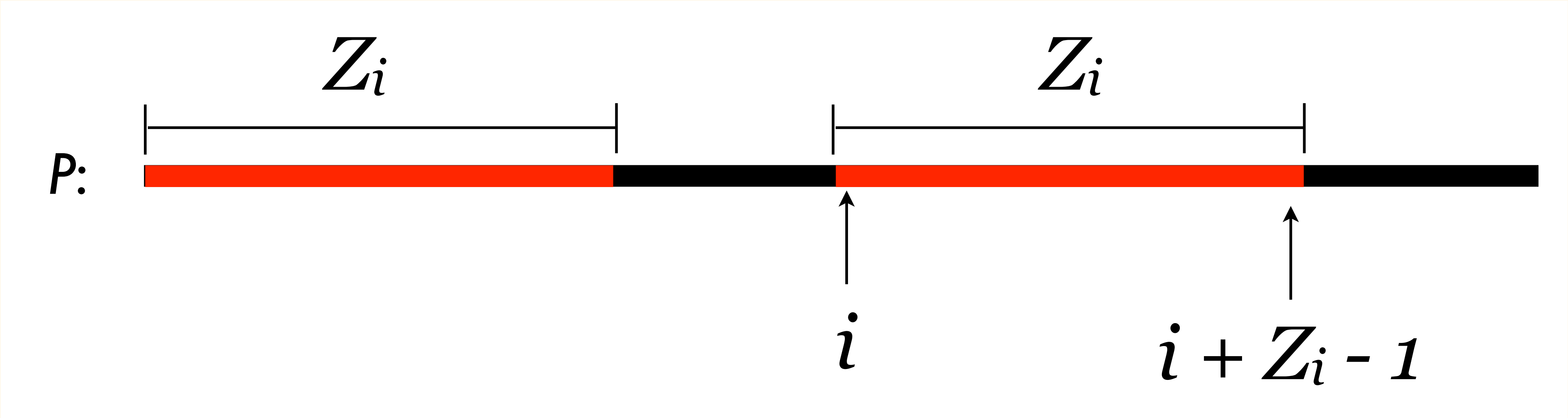
$Z_4(P) = 4$

P = photophosphorescent

$Z_6(P) = Z_{10}(P) = 3$

These regions of prefix-overlap are called *z-boxes*.

P = photo<u>pho</u>s<u>pho</u>rescent



https://www.cs.umd.edu/class/fall2011/cmsc858s/Lec02-zalg.pdf

Why do we care?

Let's stick *P* and *T* together with a sentinal character:

P$T = pho$photophosphorescent

p	h	o	$	p	h	o	t	o	p	h	o	s	p	h	o	r	e	s	c	e	n	t
0	0	0	0	3	0	0	0	0	3	0	0	0	3	0	0	0	0	0	0	0	0	0

Why do we care?

Let's stick P and T together with a sentinal character:

$$P\$T = \text{pho\$photophosphorescent}$$

p	h	o	$	p	h	o	t	o	p	h	o	s	p	h	o	r	e	s	c	e	n	t
0	0	0	0	3	0	0	0	0	3	0	0	0	3	0	0	0	0	0	0	0	0	0

Any point whose $Z_i = |P|$ matches!

How to calculate Z_i?

Naïve way: O(nm)

There exists a linear-time solution!

Next steps:

Knuth-Morris-Pratt:

A linear-time extension to what we've seen, with some clever preprocessing to allow larger shifts.

Boyer-Moore:

A totally different approach, moving right-to-left and often achieving sub-linear performance.