A Representation and Reasoning System (RRS) is made up of:

- Syntax of Datalog
- Semantics of Datalog
- Adding Variables to Semantics
- Models
- Logical Consequence
- Two Views of Semantics
  - Logical Consequence
  - Models
  - Addressing Equivalence to Semantics
  - Semantics of Datalog
  - Syntax of Datalog

Implementation of an RRS

- Reasoning procedure
  - Resolves nondeterminism of reasoning theory

Previous Class

- Introduced a task domains: robot delivery and wiring
- Introduced the symbolic approach
  - Symbols have meaning to the knowledge engineer
  - Symbols used to build a knowledge base that can be reasoned about
  - Facts about the world
  - Rules about the world

Computer reasoning with the facts and rules to make new conclusions:

- Proved sound but incomplete
- Fixed sound and complete
- Improved the symbolic approach
- Introduced a task domain: robot delivery and wiring

Overview
Overview

- Representation and Reasoning System
- Syntax of Datalog
- Semantics of Datalog
- Adding Variables to Semantics
- Semantics of Disjunction
- Syntax of Disjunction

Simplifying Assumptions of Initial RRS

- Each individual can be given a unique name
- Only a finite number of individuals of interest in the domain
- The environment is stable
- An agent's knowledge base consists of definite and positive
- Individuals and relations among individuals
- An agent's knowledge can be truthfully described in terms of

Choose the simplest RRS possible for your application

The richer the syntax, the more difficult the reasoning procedure

Different RRS'sgood for different domains
Knowledge Base

- male(william)
- male(george)
- female(sally)
- father(william,george)
- father(george,sally)

Example

- What are the constants?
- What are the predicate symbols?
- What are the variables?

Whether knowledge base is correct depends on semantics

- Syntax allows us to write sentences about the world
- Whether sentences are true or not depends on what the symbols mean, which will be specified by the semantics

More Syntax of Datalog

- Variable - starts with upper-case letter
- Constant - starts with lower-case letter or is a sequence of digits (numeric)
- Predicate symbol - starts with lower-case letter
- Term - either a variable or a constant
- Atomic symbol (atom) - of the form p(t1, ..., tn) where p is a predicate symbol and t1, ..., tn are terms
Interpretation

An interpretation is a triple $I = (D, \phi, \pi)$ where

- $D$ the domain, is a nonempty set. Elements of $D$ are individuals
- $\phi$ maps each constant to an element of $D$, $\phi(c)$ denotes individual $c$.
- $\pi$ maps each n-ary predicate symbol to a subset of $D^n$. Alternatively, $\pi$ can think of $\pi$ as mapping each tuple in $D^n$ to true or false.

**NOTE:** An interpretation is not a subset of constants.

Semantics

Semantics concerns two things

- Set of individuals in the domain, and relations between them
- How constants and predicate symbols in the syntax correspond to the individuals and relations in the domain

We call this an interpretation:

- Individuals and relations in the domain
- How constants and predicate symbols in the syntax correspond

Two views of semantics

- Logical Consequence
- Model Checking

Syntactic Semantics

Syntactic Semantics of Datalog

Representing and Reasoning Syntactic Semantics
Example: (focus on all interpretations, not just intended one)

- Language with constants \( a \) and \( b \) and 1-ary predicate \( \text{female} \)
- Domain with \( D = \{ x, y, z \} \)

How many different \( \varphi \)'s?

\[
\begin{align*}
\varphi_1(a) \\
\varphi_2(b) \\
\varphi_3 \varphi_4 \varphi_5 \varphi_6 \varphi_7 \varphi_8 \varphi_9
\end{align*}
\]

Example Continued

- William and George are male, Sally is female
- Let's have \( \pi \) map male to \( \{ < William >, < George > \} \)
- female to \( \{ < Sally > \} \)
- Knowledge Engineer decides on mapping of predicates
  - Must decide on the mapping for all predicates
  - Hence, must do mapping for \( \text{male} \) even if no facts in \( \text{KB} \) about \( \text{male} \)

This is an example of an intended interpretation:

- The interpretation that the knowledge engineer has in mind when coming up with the language and knowledge base

Example Interpretation: Robot

- \( D \) is the set of people
- It is the actual people, not the names
- \( \varphi \) maps constants of syntax to objects in the domain
  - \( \varphi(\text{william}) = \text{William} \)...
- Knowledge Engineer decides \( D \) and mapping of all constants to \( D \)
Semantics of Connectives

Still need to specify what \( \land \) and \( \leftarrow \) mean.

\[
\begin{array}{|c|c|c|}
\hline
p & q & p \land q \\
\hline
\text{true} & \text{true} & \text{true} \\
\text{true} & \text{false} & \text{false} \\
\text{false} & \text{true} & \text{false} \\
\text{false} & \text{false} & \text{false} \\
\hline
\end{array}
\]

- \textit{Nota bene!}
  - \( p \leftarrow q \) is true when both \( p \) and \( q \) are false.
  - \( p \land q \) doesn't always correspond to 'english' meaning.

Thus \( h \leftarrow b_1 \land ... \land b_m \) is false in \( I \) if \( h \) is false in \( I \) and each \( b_i \) is true in \( I \).

Semantics of \( \land \) and \( \leftarrow \) part of Datalog

Determining Truth of Ground Atoms

- Ground atom has no variables.
- \( p(t_1, ..., t_n) \) maps to true if \((\phi(t_1), ..., \phi(t_n)) \in \pi(p)\)
  - otherwise to false.
- What does \( \text{male}(\text{george}) \) map to?
  - \( \phi(\text{george}) = \text{George} \)
  - \( \pi(\text{male}) = \{<\text{William}>, <\text{George}>\} \)
  - \( <\text{George}> \in \{<\text{William}>, <\text{George}>\} \)

- So it maps to true.
- For predicates without arguments, \( \pi(p) \) is either the set with the empty tuple \{\} or it is empty.

Semantics of Ground Atoms comes from interpretation.

Example Continued

- For \( x \in \pi_1(\text{female}) \), \( y \in \pi_2(\text{female}) \), \( z \in \pi_3(\text{female}) \).

- How many different interpretations are there altogether?


different combinations of \( \phi \) and \( \pi \)

\[
\begin{array}{|c|c|c|}
\hline
\text{How many different interpretations are there altogether?} \\
\hline
\text{not a line} & \text{not a line} & \text{not a line} \\
\hline
\text{not a line} & \text{not a line} & \text{not a line} \\
\hline
\text{not a line} & \text{not a line} & \text{not a line} \\
\hline
\text{not a line} & \text{not a line} & \text{not a line} \\
\hline
\end{array}
\]
Semantics & Variables

- How do we interpret clauses such as
  \[ \text{person}(X) \leftarrow \text{female}(X) \]
- Clause is true if it is true for all values of \( X \)
  - \( \text{person}(X) \) must be true whenever \( \text{female}(X) \) is true
- Remember, knowledge engineer had to specify mapping for all predicates, even room
  \[ \pi(\text{female}) \subseteq \pi(\text{person}) \]
- It really has a universal quantifier
  \[ \forall X: \text{female}(X) \leftarrow \text{person}(X) \]
- So, variables have an implicit universal quantifier over the clause

Overview

- Representation and Reasoning System
- Syntax of Datalog
- Semantics of Datalog
  - Adding Variables to Semantics
  - Models
  - Logical Consequence
  - Two Views of Semantics
    - Semantic vs. Datalog
    - Syntactic vs. Datalog
    - Representation and Reasoning Sytem

Limitations of Datalog

- More expressive knowledge can include negative knowledge
- In a knowledge base fact, each fact is independent of the other
- Cannot write a rule that ensures just one of \( \text{male} \) and \( \text{female} \) is true for any person
- Even if every person is male or female, both predicates needed
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Sets of Clauses

A set of clauses is true in an interpretation if each clause is true in the interpretation.

- Note that we universally quantify for the variables over each clause.

\[
\begin{align*}
\text{person}(X) & \leftarrow \text{male}(X) \\
\text{parent}(X,Y) & \leftarrow \text{father}(X,Y) \\
\text{grandfather}(Z,X) & \leftarrow \text{father}(Z,Y) \land \text{parent}(Y,X)
\end{align*}
\]

Overview

- Representation and Reasoning Systems
- Logical Consequence
- Models
- Adjoin Equivalents to Semantics
- Semantics of Datalog
- Syntax of Datalog
- Two Views of Semantics

Variable Assignment: Formal Definition

- Define a variable assignment \( \rho \) that maps each variable to some object in the domain.
- Together with an interpretation, \( \phi \), we can say:
- \( \phi \) maps each atom to some object in the domain.
- \( \rho \) maps each variable to some object in the domain.
TWO VIEWS OF SEMANTICS

- Logical Consequence
- Models

Adding Variables to Semantics

Semantics of Datalog

Syntactic vs. Semantic

Representation and Reasoning Sytem

Example with constants

- Example:
  - Focus on all interpretations, not just intended one

+ Language with constants $a$ and $b$ and 1-ary predicate $\text{girl}(\_)$
+ Domain with $D = \{x, y, z\}$
+ 9 $\pi$'s and 8 $\phi$'s, so 72 interpretations

- How many models of $\text{KB} = \{\text{girl}(a), \text{girl}(b)\}$?
  - Checking each would take too long, so let's break down into subcases
    - Case 1: $\phi_i(a) = \phi_i(b)$
      - How many of the 9 $\phi_i$'s have $\phi_i(a) = \phi_i(b)$?
        - When $\phi_i(a) = x$ and $\phi_i(b) = y$, which $\pi$'s make KB true?
      - So how many models with $\phi_i(a) = \phi_i(b)$?
    - Case 2: $\phi_i(a) \neq \phi_i(b)$
      - How many of the 9 $\phi_i$'s?
        - When $\phi_i(a) = x$ and $\phi_i(b) = y$, which $\pi$'s make KB true?
      - So how many models with $\phi_i(a) \neq \phi_i(b)$?

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Models
Overiew

• Representation and Reasoning System
• Syntax of Datalog
• Semantics of Datalog
• Adding Variables to Semantics
• Semantics of Datalog
• Syntax of Datalog
• Representation and Reasoning System

Example Revisited

• KB:
  • p ← q.
  • q.
  • π(p)
  • π(q)
  • π(p ← q)

I_1: TRUE TRUE
I_2: TRUE FALSE
I_3: FALSE TRUE
I_4: FALSE FALSE

• Does KB |⇒ p?

Logical Consequence

• If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written KB |⇒ g, if g is true in every model of KB.

- This means that our KB, by its definition, always forces g to be true.
- Other terms that mean the same thing:
  - g logically follows from KB
  - KB entails g

• KB |⇒ g if there is no interpretation in which KB is true and g is false.

• KB |⇒ g if g is not a logical consequence of KB.
Summary of Semantics

- User has intended interpretation
  - But just tells the computer a small set of facts that hopefully adequately captures the user's intended interpretation
- Computer answers true if all interpretations that make the knowledge base true (models) also make the question true
- Does not necessarily entail the user's intended interpretation
- But just tells the computer a small set of facts that hopefully capture the intended interpretation

Computer's View of Semantics

- Computer given the knowledge base
- Computer doesn't have access to the intended interpretation
- User asks it a question
  - Computer should answer true if the knowledge base |= q
  - Otherwise, computer should answer "I don't know"
  - There is at least one model in which q is false
- Note q might have been true in user's intended interpretation. In this case, user didn't have enough clauses in the knowledge base to sufficiently narrow down the models

Aside: computer could answer the question by enumerating over all of the possible interpretations (model checking)
- But number of interpretations grows quickly!!

User's View of Semantics

- Choose a task domain: intended interpretation
- Associate constants with individuals you want to name
- For each relation you want to represent, associate a predicate symbol in the language
- Tell the system clauses that are true in the intended interpretation: axiomatizing the domain
- Hopefully you tell it enough knowledge about the domain so that it can conclude everything you want it to
- Ask questions about your domain

Choose a task domain: intended interpretation