Chapter 2: CFG

Question 1: Question 2.14

Question 2: Question 2.15

This question is not well-written. Consider if we add the rules \( S \rightarrow SS \mid \epsilon \). Does this properly generate the star of the language?

Question 3: Question 2.16

In textbook question 2.2, you already proved that CFL’s are closed under union. So, just do concatenation and star.

Chapter 2: PDA

Question 4

Give a state diagram for a PDA that ensures that the first and last character of the string is the same (Question 2.4 b), the string has length at least 2, and the alphabet is \{0,1\}.

Also give a DFA (deterministic finite automata). Although you could give the same answer for both (as a DFA is a trivial PDA), make sure your PDA makes use of the stack in a meaningful way, and so should have fewer states than your DFA.

Question 5: Question 2.10

See the answer for Question 2.7 on page 132 for the level of detail expected in an ‘informal description’.

Question 6: Question 2.11

Show Languages are CF by constructing a CFG or PDA

Question 7: Question 2.25

Question 8: Question 2.20

In homework 4, you proved this where A is regular and B is regular. Make sure that you thoroughly understand the answer.

Do this proof by using a PDA for language A, and a DFA for language B. You will construct a PDA for \( A/B \).
Prelude to Pumping Lemma

Question 9: Question 2.35

This is a simpler version of the question in the textbook. In the next four steps, you will show that if $G$ generates a string $s$ with a derivation having at least $2^b$ steps, $s$ has a parse tree of height at least $b + 2$.

**Part a**

First, prove that a string $s$ has $|s| * 2 - 1$ steps in its derivation.

**Part b**

Instead of focusing on a sequence of derivations, we can arrange the derivations into a parse tree. A parse tree has nodes that are variables, and its leaves are the terminals. Since $G$ is in Chomsky normal form, each node will either go to one leaf, or to two other nodes. The shortest parse tree has a height of 2 (start node goes to a terminal node).

For the following grammar, the parse tree will be very skinny.

$S \rightarrow AS|a$
$A \rightarrow a$

How high will the parse tree of the string $a^n$ be?

**Part c**

Rather than thinking of really skinny parse trees, think of how bushy they can be. This of course will depend on the grammar.

We want to determine how short a parse tree can be for a string, for any grammar. In other words, we want to determine a minimum bound.

What is the longest string that a parse tree of height 2, 3, 4, 5 can generate? How about a parse tree of height $n$?

**Part d**

Now argue that if $G$ generates a string $s$ with a derivation having at least $2^b$ steps, $s$ has a parse tree of height at least $b + 2$. 